Cross-Country Comparisons of Student Sense Making: The Development of a Mathematics Processing Framework

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This paper identifies the strategies Singaporean and Australian students (n = 1,187) employed to solve a 24-item mathematics test. A mathematics-processing framework is proposed, which describes the way primary-aged students successfully process graphic and non-graphic mathematics tasks. There were distinct differences in the way in which the students from the respective countries approached the tasks with the Singaporean students more likely to employ strategies that were explicitly taught and practiced in the classroom, whereas the Australian students tended to employ a more diverse range of approaches.

Singaporean students usually outperform Australian students on international comparisons of numeracy competence (e.g., TIMMS). Cross-country differences in classroom practices and lesson development have also been noted (e.g., Logan & Ho, this symposium), including the tendency for Singaporean teachers to provide more explicit instruction of problem-solving methods or heuristics (Ho & Lowrie, 2012). Much less is known about how students from different countries represent, decode and encode mathematics ideas. The focus of this paper is to propose a processing framework that can be used to monitor how primary-aged students represent, decode and decode mathematics tasks. It is argued that the framework is particularly useful when conducting cross-country and cross-cultural studies—especially across countries with distinct pedagogical practices.

Representing and Processing Mathematics Tasks

Krutetskii (1976) classified student's problem-solving solutions into three categories: *verbalizers* who preferred to process information using verbal-logical reasoning; *visualizers* who tended to prefer to use visual imagery; and *mixers*, who did not have a preference for either approach. His seminal work encouraged others to describe the division between verbal and imagery representations along a novelty-of-task dimension (e.g., Hegarty & Kozhevnikov, 1999). Kaufmann (2000) argued that great precision is achieved in a verbal-propositional description of a task. Such representations are efficiently described, even though they demand a full range of computational operations, with such processing best utilised when tasks are not complex. By contrast, tasks demanding extensive use of imagery are likely to be more ambiguous—however imagery is particularly useful where the need for processing is high, which is generally the case with novel task conditions (Lowrie & Kay, 2001).

Recent studies have found a student's use of analytic or visual-imagistic processing is independent of spatial ability (Stieff, Ryu, Dixon, & Hegarty, 2012) and most likely associated with the understanding and proficiency they bring to the task rather than a particular preference or style (Lowrie & Kay, 2001). Blazhenkova and Kozhevnikov (2009) argued that the fixation on c ognitive style research was problematic since "neuropsychological data had revealed the existence of two distinct imagery systems that encode and process visual imagery in different ways" (p. 640). From a mathematics education perspective, *object imagery* is associated with the way objects are processed in terms of shape, colour and texture or what Bertin (1983) describes as saturation. The

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spatial imagery system processes object location, transformation and other spatial relationships. These two processing systems tend to work independently of one another. It also seems to be the case that the manner in which students process information and the strategies and approaches they use to solve the tasks *are* dependent on their conceptual understanding but also the manner in which the respective tasks are represented. Moreover, representation may be influenced and even dominated by embedded elements of the task.

To date, most studies have considered students' representation and information processing when solving traditional word-based problems. As Blazhenkova and Kozhevnikov (2009) argued, the validity of verbal-visual spectrums have been questioned because they are essentially descriptive and "do not attempt to relate cognitive styles to contemporary cognitive science theories" (p. 640), since they fail to consider tasks in visual or spatial domains (Kounios & Beeman, 2008). Problem solving, particularly in the first eight years of schooling, involves much less word-based problem solving than was the case even ten years ago (Lowrie & Diezmann, 2009). Consequently, the assessment of students' mathematics understanding and proficiency needs to be considered in relation to both the representation of the mathematics task and the manner in which the problem solver represents his/her respective solution or approach.

Contextualising the Study

The processing framework is established within a cross-cultural study of 1,187 (Singaporean and Australian) Year 6 students—with more than 23,000 problem-solving approaches to 24 tasks analysed (24 items x at least 2 approaches x N). The tasks were both graphic and non-graphic in composition (12 of each). The graphic tasks included items containing diagrams, maps, number lines, line graphs and pie charts. The non-graphic tasks were composed of text only—commonly considered 'word problems'. The framework considers the related influences of task design, representation and strategy use on students' proficiency and fluency. Moreover, the processing framework provides opportunities to describe the influence particular classroom practices have on student performance—and specifically strategy processing approaches and representational prototypes in the current investigation.

Categorisation and Analysis of the Data

The framework describes the way students encoded and decoded information to produce solutions to the 24 mathematics tasks. A corresponding Mathematics Processing Instrument (MPI) supplemented the 24-item test, which encouraged the students to describe the approach they used to solve each item. Thus, data consisted of student scores on the test (correct and incorrect responses for each item) and the approach they used to solve the respective items. The approaches were classified as either *visual* (including approaches where the students employed predominately visual, concrete-pictorial or gestural approaches) or *nonvisual* (where the approach predominately contained algorithms, number sentences or pre-algebraic reasoning). Several multivariate procedures were used to analyse these data to complement the rich qualitative analysis described elsewhere in the symposium (see Greenlees; Ho & Logan; Logan & Ho, this symposium).

The Development of a Mathematics Processing Framework

In terms of the non-graphic tasks, the participants frequently decoded information using one or more heuristics (e.g., constructing number sentences, using symbols, drawing diagrams as models). Generally, the Singaporean students' decoding techniques were efficient and followed template-like, worked sample, designs. For the most part, the Singaporean students' solutions were indistinguishable in terms of representation—indicating the influence direct teaching instruction had on the way the students decoded the tasks. This was also the case when students produced incorrect solutions, that is, the representational structure was similar to that of students who produced correct solutions—the difference(s) were commonly associated with calculation errors. By contrast, the Australian students' approaches to non-graphic tasks were more idiosyncratic, detailed and contained more non-essential information. Correct solutions included a number of variations on a common approach or method. It was evident the students were not utilising specific representations of an heuristic in these successful approaches. Unlike the Singaporean students, incorrect solutions tended to contain no logical sequencing of ideas. For the most part, these students used algorithms or computational procedures inappropriately such as selecting the wrong operation or not understanding which information was pertinent and which information was redundant.

With respect to the graphic items, the students tended to use encoding techniques (including visualisation and concrete imagery) to (re)present the task before utilising decoding techniques. This finding was surprising—with students wanting to construct their own images and representations of the tasks despite the fact visual and graphic representations were already contained within these tasks. Even though the tasks contained graphic stimulus that had to be decoded, the students frequently produced their own representations as part of the solution. That is, for the graphic tasks, students frequently used decoding skills to interpret the graphic information whilst also using encoding techniques to produce images (either on paper or in the mind's eye) to help organise information and potentially scaffold understandings.

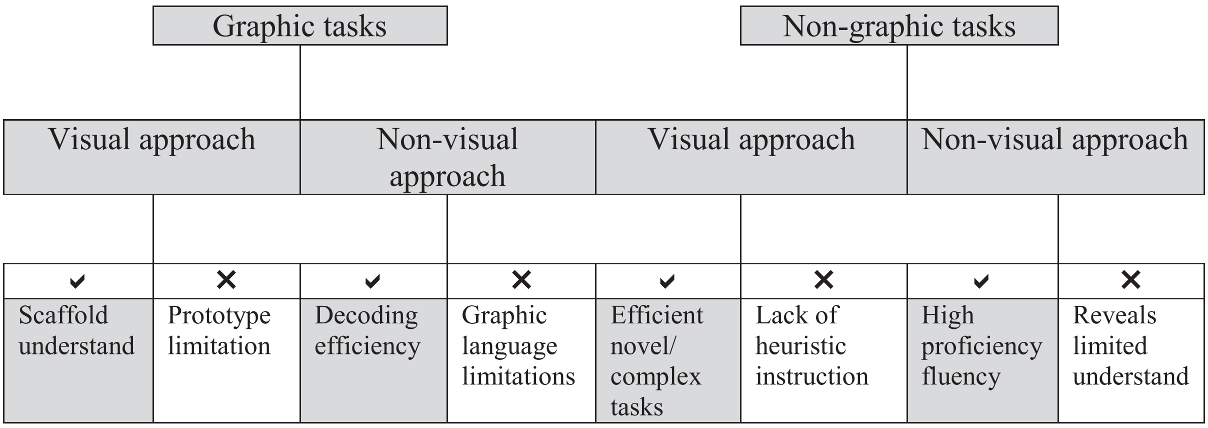

Figure 1. Processing graphic and non-graphic mathematics tasks.

Figure 1 provides an illustration of the proposed *mathematics-processing framework*. The framework divides mathematics tasks into two representational modes—graphic and non-graphic tasks. Additionally, the framework categorises student processing into visual and non-visual representations for each of the two representational modes. Correct (✔) and incorrect (✕) responses for the two processing modes (i.e., visual and non-visual) describe the central mathematics elements that are derived from such effective or ineffective processing. The common approach for each category is displayed.

The Australian students tended to utilise visual strategies (successfully) more frequently than Singaporean students to solve graphic tasks. They were more inclined to ‘draw on’ the graphics embedded within the graphic tasks—the Singaporean students seldom did this. Noteworthy, the Australian students’ performance on the 12 graphic tasks was much closer to the mean scores of the Singaporean students than was the case with the non-graphic items. When solving the non-graphic items, the Singaporean students were much more likely to employ successful visual strategies than the Australian students. It was evident they possessed a greater repertoire of heuristics (including drawing diagrams) to solve these word-based tasks. As Ho and Lowrie (2012) reported, Singaporean students utilise effective visual approaches (e.g., the model method) to solve non-graphic tasks—with such approaches explicitly taught in the classroom. For the non-graphic tasks, and especially the most difficult of these tasks, a high proportion of Australian students used ineffective non-visual strategies to solve the tasks. For the most part, the students used algorithms or computational procedures inappropriately, for example selecting the wrong operation or not understanding which information was pertinent and which was redundant.

Conclusion

This model provides insights into primary-aged students’ mathematical understanding and proficiency in relation to task representation. Unlike other models that have been proposed in the literature, this framework is intended to describe how students solve mathematics tasks across both non-graphic and graphic representations. Moreover, the model provides strong insights into students’ thinking and conceptual understanding through a representational lens. The framework has potential for assessing students’ mathematics development and could be used as a dynamic assessment tool. Importantly, it allows teachers to include task representation as another source of information to describe students’ sense making and gain insights into students’ proficiency.

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